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EFFECT ON STABILITY OF THE AXIAL ROTATION OF A MISSILE CONTAINING A LIQUID MASS

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Jean-Pierre Chautard

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EFFECT ON STABILITY OF THE AXIAL ROTATION OF A MISSILE CONTAINING A LIQUID MASS

Jean-Pierre Chautard

(Research Engineer, O.N.E.R.A.)

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SUMMARY

The axial rotation of a missile containing a considerable liquid mass may produce stability perturbations.

In fact, experiments have shown that there may be resonance between the perturbed rotation of the liquid and the lateral oscillation of the missile.

Means of avoiding this effect, though difficult to apply in practice, are proposed.

Experiments on the initiation of fluid motion and on the process of tank emptying are also described.

I. INTRODUCTION

Axial rotation of a missile not subject to automatic control may appear spontaneously owing to structural dissymmetry or may be deliberately induced either to distribute parasitic forces in all directions or to provide a stability that would otherwise be lacking. It is necessary to know what effect this rotation has on missiles containing a considerable mass of liquid.

The present study is essentially experimental. I shall examine in turn the initiation of fluid rotation in the tank, resonance between the perturbed rotation of the liquid and the lateral oscillation of the missile, and the conditions of stability. The report concludes with a description of certain preliminary experiments on the problem of fuel supply.

The results obtained on the various pieces of apparatus are applicable to a real missile, since the Reynolds and Froude numbers of the fluid motion are preserved. Thus, the linear scale for the tank and the time scale are defined on the basis of the acceleration ratio

and the kinematic viscosity ratio of the liquid. If Γ denotes the acceleration due to the thrust alone, the former is defined by:

$$\frac{}{L} = \left[\frac{\Gamma}{g}\left(\frac{\nu}{N}\right)^2\right]^{1/3} \text{ and the latter by, } \frac{t}{T} = \left[\frac{\Gamma}{g}\ \frac{1}{L}\right]^{1/2}.$$

Moreover, the solid part of the missile, which is not represented exactly, must have its own period of lateral oscillation, derived from that of the model by means of the above ratio of similitude.

All the experiments were made with mercury, the low kinematic viscosity of which gives an acceptable linear scale. Thus, for example, the scale of a model of a hybrid rocket with a nitric acid propellant and an acceleration Γ equal to 2 g, will be 1/3, and the time ratio $\sqrt{2/3}$.

II. INITIATION OF FLUID ROTATION

As shown in Fig. 1, which represents the shape of the free surface, in the transient regime, of the mercury contained in a tank 100 mm in diameter rotating at 400 rpm, the initiation of fluid rotation is quite rapid. The paraboloid that constitutes the equilibrium surface in the steady state may be regarded as fully formed after only 6 minutes of rotation of the tank.

This result is very different from that given by the theory of the rotation of a liquid in a tank of infinite height. This discrepancy follows from the fact that for a tank of finite height the rotation of the liquid is modified by axial and radial circulation and, at least in part, is brought about by convection. This phenomenon has been studied by Professor Bouasse [1].

Finally, in keeping with this result, it is possible to obtain /4 a considerable reduction in the time of initiation of rotation by replacing the smooth bottom with a rough or bladed one.

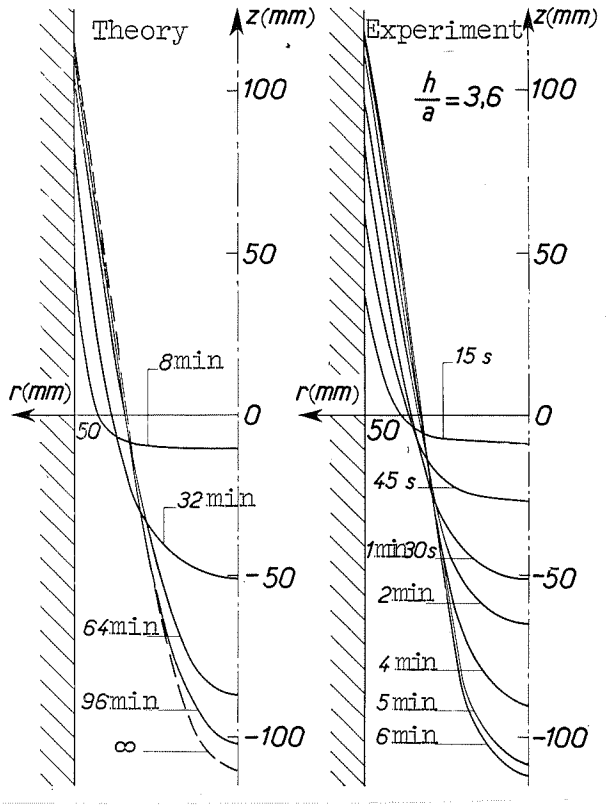


Fig. 1. Section through free surface of liquid (rotation frequency of tank = 6.67 revolutions per second).

III. ROLL-PITCH RESONANCE

III.1 First experiments

The first experiments on the effect on stability of the rotation of the liquid in a missile tank were performed on the so-called "roll-pitch" apparatus illustrated in Fig. 2. A cylindrical tank of variable height and 100 mm in diameter, rotating about its axis, was free to oscillate about an axis normal to its own located in the plane of the bottom. Two interchangeable springs, arranged symmetrically with respect to the axis, provided the restoring couples.

The first experiments show that at certain rates of rotation in roll, which I shall call critical speeds, in response to a pitching impulse, despite the considerable damping due to the apparatus, the pitch oscillations are amplified or maintained without change in frequency.

This phenomenon I shall call roll-pitch resonance.

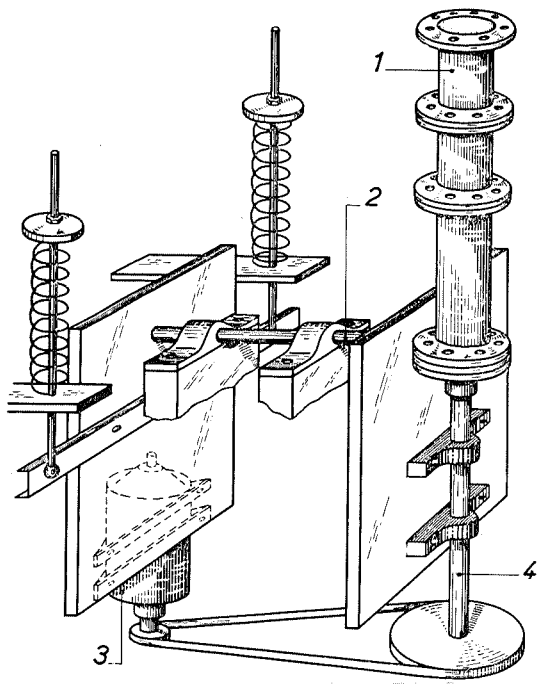


Fig. 2. Diagram of roll-pitch apparatus.

1 - tank, 2 - axis of oscillation, 3- motor, 4 - axis of rotation.

Since the rotation frequency is then always very much greater than the pitch frequency, there can be no question of resonance due to parasitic forces connected with the tank, on the contrary, the effect is due solely to the presence of the liquid.

The critical speeds form a fairly extensive, continuous scale. Thus, for example, if the tank is filled with mercury up to a height of 200 mm and the natural frequency of the lateral oscillations is

2.85 cps, which corresponds to 2.3 cps for the hybrid missile defined above (Section I), resonance occurs at rotation frequencies lying between 4.7 and 9.7 rps, or between 3.8 and 7.9 rps for the actual missile. However, at the lowest value of the rotation frequency, the effect can only be provoked after 5 minutes of rotation and ceases in 10 seconds or so. Only at frequencies of more than 6.5 rps (5.3 rps) does the effect become permanent, and only at frequencies greater than 8 rps (6.5 rps) can it be induced within the first 10 seconds of rotation.

Furthermore, these critical speeds vary slightly with the natural frequency of pitch, the height of the liquid in the tank, and the roughness of the bottom. In particular, if the bottom is fitted with radial blades, the effect becomes permanent at lower critical speeds. In the various experiments these always lay between 1.25 and 5 times the natural frequency of pitch.

Finally, films and stroboscopic observations of the phenomenon reveal a wave propagated, without apparent deformation, at the surface of the liquid, along the wall, with a frequency of rotation equal to the frequency of pitch and in phase with the latter (Fig. 3).

III.2 Interpretation of the effect

From all these experiments it appears that resonance is due to the propagation of a perturbation about the axis of the tank, at the natural frequency of and in phase with the pitch oscillations. Moreover, the propagation velocity of such a perturbation must depend on the state of the tangential velocities of the liquid in rotation. Thus, resonance can only occur for a certain distribution of the velocities within the whole or a part of the liquid. /5

This explains, in particular, why in the case of the higher critical speeds of rotation the effect can only develop during the period of initiation of fluid rotation.

Moreover, the energy needed to maintain or amplify the pitch oscillations is supplied by the liquid, at the expense of its kinetic energy of rotation. Consequently, if the force applied to rotate the liquid is insufficient, we get degradation of the velocity state of the fluid, and the propagation frequency is no longer equal to the natural pitch frequency. Thus, the process is interrupted and will not resume until the liquid has recovered its initial velocity state.

This is what happens at the lower critical speeds; however, if the drive is improved, even at these speeds the effect will become permanent.

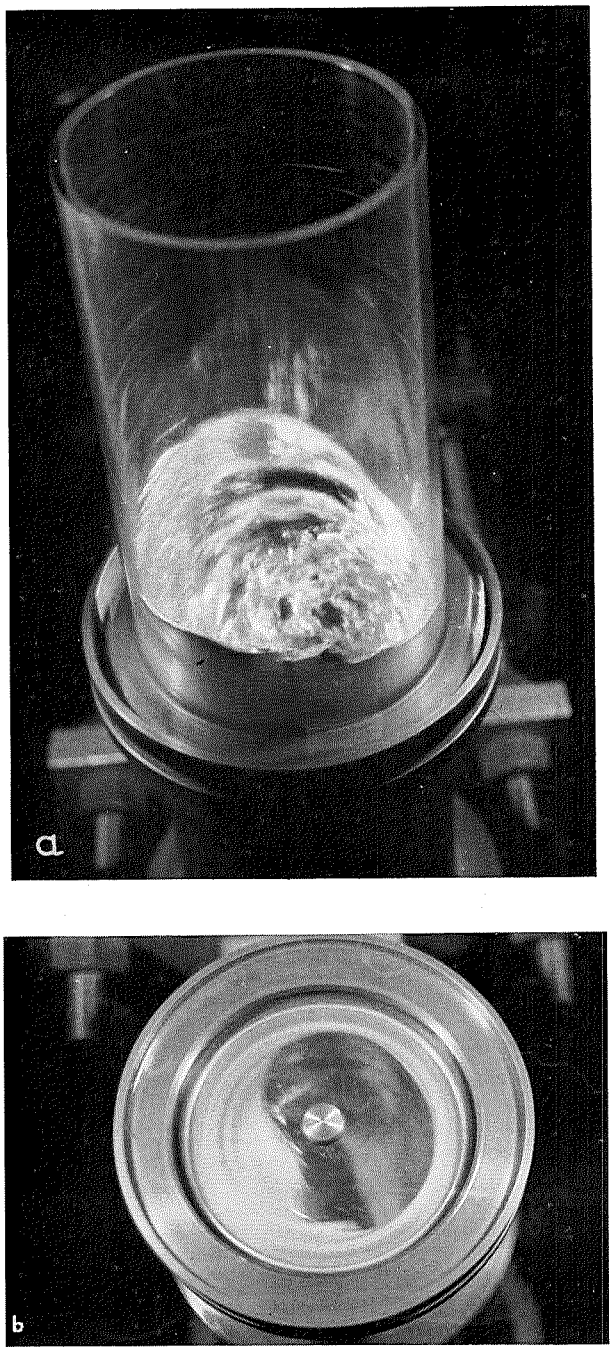


Fig. 3. Photograph of free surface of liquid during roll-pitch resonance.

III.3 Calculation of the propagation velocity

The rate of propagation of the perturbation can be calculated if we make the following simplifying assumptions:

- The cylindrical talk of radius a is made to rotate at a constant velocity ω_0 about its fixed axis Oz.

- The liquid is a perfect fluid and rotates as a unit with the tank.

- The acceleration due to gravity is negligible as compared with these centrifugal accelerations. The liquid is thus distributed in a cylinder of inside radius ka.

- Finally, the small perturbations about this configuration, investigated here, are independent of z.

From the latter assumption it follows that the Coriolis inertia forces, to which the liquid is subjected in its relative motion with respect to the tank, depend on a potential: - U. In fact, being in a plane flow, the Coriolis acceleration may be expressed in terms of the stream function ψ, thus:

$$\vec{\Gamma}_c = 2\vec{\omega_0} \wedge \vec{V}_r = 2\omega_0 \overrightarrow{\text{grad } \psi}.$$

Moreover, since the driving acceleration is derived from the potential: $-\omega_0^2 \frac{r^2}{2}$, and the mass forces reduce to the driving and Coriolis inertia forces, the relative velocities of the liquid depend on a potential φ (r, η, t), a conjugate harmonic function of ψ. /6

Thus, in cylindrical coordinates, the system of equations of motion of the liquid may be written (Fig. 4):

$$\text{Continuity} \quad \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \eta^2} = 0 \qquad (1)$$

Motion $$\frac{\partial \varphi}{\partial t} + \frac{p}{\rho} - \omega_0^2 \frac{r^2}{2} + U = C^{te} \qquad (2)$$

Wall $$\left(\frac{\partial \varphi}{\partial r}\right)_{r=a} = 0 \qquad (3)$$

Free surface $$\left(\frac{dp}{dt}\right)_{r=ka} = 0 \qquad (4)$$

with $$\overrightarrow{\text{grad } U} = 2\omega_0 \overrightarrow{\text{grad } \psi} = -2 \frac{\omega_0}{r} \frac{\partial \varphi}{\partial \eta} \vec{n} + 2\omega_0 \frac{\partial \varphi}{\partial r} \vec{t}. \qquad (5)$$

Bearing in mind (3), the solution of equation (1) may be written:

$$\varphi = \sum_{m=1}^{\infty} A_m \cos (m\eta - \Omega_m t)\left[\left(\frac{r}{a}\right)^m + \left(\frac{r}{a}\right)^{-m}\right] \qquad (6)$$

where the A_m are constants depending on the initial conditions and the Ω_m are constants to be determined by the free surface condition (4).

In fact, upon substituting for φ, equation (2) gives:

$$\frac{p}{\rho} = \omega_0^2 \frac{r^2}{2} - \sum_{m=1}^{\infty} A_m \sin (m\eta - \Omega_m t)\left\{ \Omega_m \left[\left(\frac{r}{a}\right)^m + \left(\frac{r}{a}\right)^{-m}\right] \right.$$

$$\left. + 2\omega_0 \left[\left(\frac{r}{a}\right)^m - \left(\frac{r}{a}\right)^{-m}\right] \right\} + C^{te}. \qquad (7)$$

Transferring this value of p to the free surface condition (4) and neglecting terms of the second order in $A_i A_j$, we get, for each value of m:

$$m\omega_0^2 (k^m - k^{-m}) + 2\omega_0\Omega_m (k^m - k^{-m}) +$$

$$\Omega_m^2 (k^m + k^{-m}) = 0. \qquad (8)$$

Furthermore, in the axes OX, OY tied to the tank and for unit height, the force exerted by the liquid on the lateral wall:

$$\vec{F} = a \int_0^{2\pi} p_p (\cos \eta \, \vec{I} + \sin \eta \, \vec{J}) \, d\eta.$$

Replacing the pressure at the wall p_p by the value derived from equation (7), the above integral is zero for all values of m other than 1, and we get:

$$\vec{F} = 2 \pi\rho A_1\Omega_1 \left[\cos \left(\Omega_1 t - \frac{\pi}{2}\right) \vec{I} + \sin \left(\Omega_1 t - \frac{\pi}{2}\right) \vec{J}\right]. \qquad (9)$$

Thus, the perturbation creates a rotational force with the relative angular velocity:

$$\Omega_1 = \omega_0 \frac{1 - k^2 \pm \sqrt{2 (1 - k^2)}}{1 + k^2} \qquad (10)$$

or the absolute angular velocity, which I shall call the propagation

velocity of the perturbation:

$$\omega = \omega_0 + \Omega_1 = \omega_0 \frac{2 \pm \sqrt{2\,(1 - k^2)}}{1 + k^2}$$

introducing the filling factor, ratio of liquid volume to tank volume, $\lambda = 1 - k^2$:

$$\omega = \omega_0 \frac{2 \pm \sqrt{2\lambda}}{2 - \lambda}. \tag{11}$$

The experimental investigation (Section III.4) shows that only the solution corresponding to the minus sign needs to be considered.

Finally, equating its radial rate of displacement with that of the particle passing through the point, the expression for the free surface is:

$$r_{SL} = ka + \int_0^t \left(\frac{\partial \varphi}{\partial r}\right)_{r=ka} dt + \zeta_0.$$

In particular, the first term of the perturbation gives the conchoid of a circle (Fig. 4):

$$r_{SL} = ka + \frac{1 - k^2}{ak^2} A_1 \frac{1}{\Omega_1} \sin(\eta - \Omega_1 t). \tag{12}$$

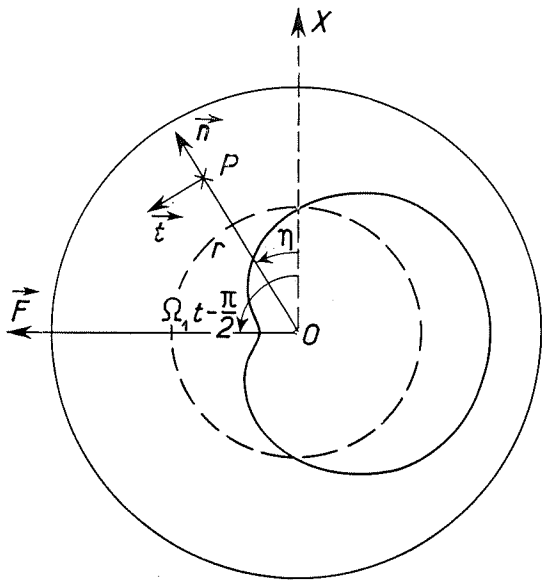


Fig. 4. Theoretical shape of free surface of liquid and coordinate system.

III.4 Experimental verification

This theory of roll-pitch resonance is difficult to verify experimentally, since the calculations are only valid if the axis of the tank is fixed. Thus, the lateral oscillation of the tank must be confined to small vibrations. In the apparatus selected (Fig. 5a, 5b), the tank, 145 mm high and 50 mm in diameter, rotating axially at high speed, is placed on an oscillating table. The lateral oscillation is heavily damped by felt pads; recovery is provided for by means of elastic blades. /7

If the above theory is correct, the oscillation frequency of the table during spontaneous resonance should be equal to the propagation frequency of the perturbation. Now, Fig. 6, which gives the distribution of the experimental points in relation to the curve of variation of the ratio of the angular velocity of propagation of a perturbation in the liquid to the angular velocity of the tank ω/ω_0 as a function of the filling factor λ, shows the excellent agreement between theory and experiment for values of ω/ω_0 less than 1.

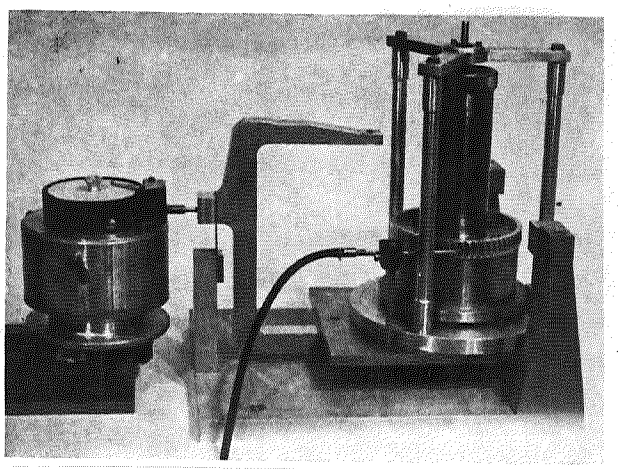


Fig. 5a. Apparatus employed to verify theory of roll-pitch resonance.

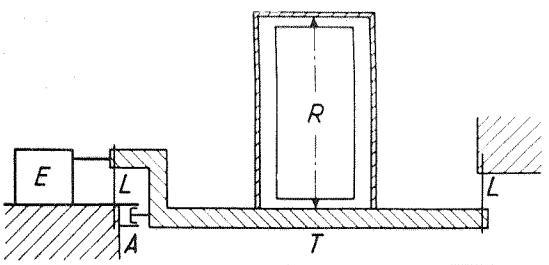
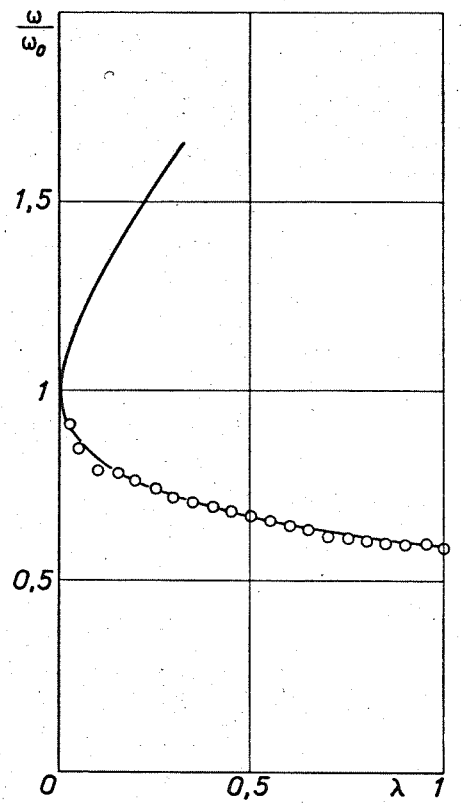


Fig. 5b. Principle of apparatus employed to verify theory of roll-pitch resonance:

R - tank, L - elastic blades, T - oscillating table, A - damper, E - recorder for registering movements of table.

On the other hand, as with the experiments on the roll-pitch apparatus, it was not possible to obtain experimental points such that the oscillation frequency of the tank is greater than its frequency of rotation. Thus, it appears that the perturbation at $\omega/\omega_0 > 1$, i.e.,

for positive Ω_1, which could not be eliminated mathematically, does not occur in reality.



[commas represent decimal points]

Fig. 6. Ratio of pulsation of later oscillation to angular velocity of tank as a function of the filling factor.

—— theoretical curve;
o experimental points.

III.5 Supplementary experiments

Supplementary experiments, performed on the roll-pitch apparatus, revealed the complete ineffectiveness of various devices designed to prevent resonance, whether connected with the tank, such as wall rings, bottom rings, bottom spirals, radial bulkheads and central tubes, or with the free surface, such as surface particles or even a piston. In fact, the majority, by improving the entrainment of the liquid, reduce the critical speed of the permanent effect.

On the other hand, these experiments also showed that the liquid is not seriously entrained if radial blades are made to rotate at the bottom of the tank in the opposite direction to the axial rotation, and that under these conditions there is no danger of resonance. This device might therefore be considered for a missile, but it requires a fairly considerable on-board power supply, and in particular very reliable control to ensure that one of the two modes of entrainment of the liquid does not prevail over the other.

IV. STABILITY OF THE MISSILE

IV.1 Sufficient condition of stability

The above experiments reveal the danger of certain rates of axial rotation for missiles containing a liquid mass subject to the action of an aerodynamic restoring couple. However, V. V. Rumyantsev [2] gives a sufficient condition of stability.

This condition results from the application of a stability criterion, previously established by the author, to the non-perturbed motion of a missile with a liquid mass in axial rotation, the center of gravity of which is in linear translational motion at constant speed. Unfortunately, the calculations are not rigorous in the case of a partly full tank.

However this may be, the sufficient condition of stability of the non-perturbed motion of the missile, in relation to 12 characteristic parameters of the motions of the missile and the liquid, at which Rumyantsev finally arrives, is:

$$\left(C\omega_0 - A\,\frac{g}{S}\right)\frac{g}{S} - \alpha > 0$$

where: C and A denote the axial and transverse moments of inertia of the solid part of the missile, relative to its center of gravity Gs, not necessarily that of the missile.

S, the greater of the principal moments of inertia C_L and A_L of the liquid, assumed solidified, relative to Gs.

g, the kinetic moment of the liquid in its non-perturbed motion about the axis of the missile.

ω_0, the angular velocity of axial rotation of the missile.

α, the proportionality factor between the moment, relative to the center of gravity of the solid, of the aerodynamic restoring couple and the sine of the angle between the axis of the missile and the velocity of its center of gravity.

If, moreover, the non-perturbed motion of the liquid is rotation as a unit with the missile, the above condition may be written:

$$KA \frac{C_L}{S} \omega_0^2 - \alpha > 0 \quad \text{with} \quad K = \frac{C}{A} - \frac{C_L}{S}.$$

This condition reveals the importance of the sign of K, since a missile, whether provided with restoring couples or not ($\alpha \geqq 0$), will be stable if K is positive; thus,

if: $$\frac{C}{A} > \frac{C_L}{A_L} \quad \text{and} \quad A_L > C_L$$

or if: $$C > A \quad \text{and} \quad C_L > A_L.$$

Now, in general, classical liquid rockets do not fit these definitions, and it would be necessary to devise new configurations. /8

Finally, these results can not be correlated with the above research, since the latter only gives an interpretation of the resonance effect, not the conditions of resonance in the case of a real missile. However, it does not appear that, for a missile subjected to a restoring couple, the shapes of the solid part and the liquid mass would have any effect on the existence of the phenomenon.

The same does not apply to a missile maneuvering in space and stabilized gyroscopically. In fact, assuming that the liquid and the

solid move as a unit, the axis of the missile, in response to an impulse imparting low initial velocities, oscillates with a pulsation:

$$\omega = \frac{C + C_L}{A + A_L}\,\omega_0 .$$

Now, according to the experimental results obtained, the angular velocity of this propagation is always less than the angular velocity ω_0 of rotation of the missile. Thus, if $\frac{C + C_L}{A + A_L} > 1$, there can not be resonance between the oscillating motion of the missile and the propagation of the perturbation. Now this condition holds true for $C > A$ and $C_L > A_L$.

IV.2 Experimental verification

In order to check these results, two cylindrical tanks, of different configuration, driven in axial rotation, were mounted in a gimbal suspension (Fig. 7). The speeds achieved permit the assumption that the (non-perturbed) liquid arranges itself in a cylinder inside the tank, so that the center of rotation always coincides with the center of gravity of the solid-liquid system.

Tank A, 40 mm high and with an inside diameter of 80 mm, always had a positive K with $C/A = 1.46$ and $S = C_L$, and could be subjected to the action of a restoring couple. Now, in this case, if the tank is not completely full, we again get resonance between the lateral oscillation of the system and the perturbed rotation of the liquid.

This confirms the reservation expressed above in connection with the validity of the stability condition for a partially filled tank. At the same time, if it is not subjected to the action of a restoring couple, tank A behaves like a solid gyroscope.

Tank B, on the other hand, 80 mm high and with an inside diameter of 40 mm, which had a negative K, became unstable, even when completely full, as soon as the rate of rotation reached a certain, quite low value; it then fell into an oscillating motion of high amplitude, which in certain circumstances went as far as complete inversion.

These results tend to show, as suggested by the sufficient condition of stability, that gyroscopic stabilization is possible for a missile of configuration similar to that of tank A, but is not possible

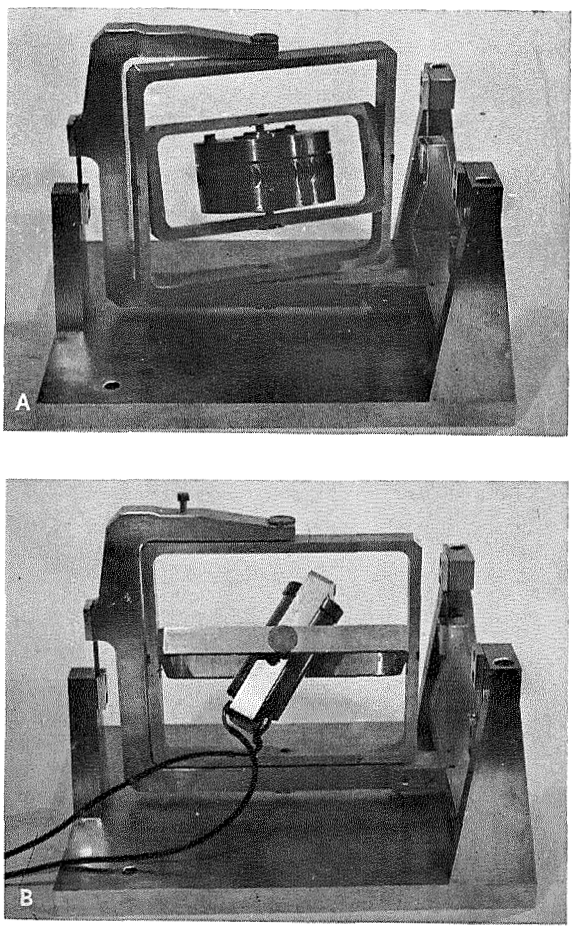


Fig. 7. Tanks mounted in gimbal suspension employed to verify sufficient condition of stability.

for a missile of configuration similar to that of tank B. However, there remains a risk of resonance between the perturbed rotation of the liquid and periodic movements of the tank, due, for example, to the vibration of the missile structure or combustion instabilities. Such a resonance is all the more dangerous in that the liquid, which is then rotating at high speed, possesses considerable kinetic energy. Moreover, the violence of the effect was revealed during the experiments by a parasitic resonance between the motion of the liquid in tank A and the oscillations of the table on which the gimbal suspension was

mounted: this resonance had a rotation frequency of the order of 40 rps.

V. FIRST RESULTS RELATING TO FUEL SUPPLY

On the roll-pitch apparatus we performed a few experiments on the emptying of a tank in axial rotation under the action of an expulsion pressure. In order to permit complete emptying, the bottom

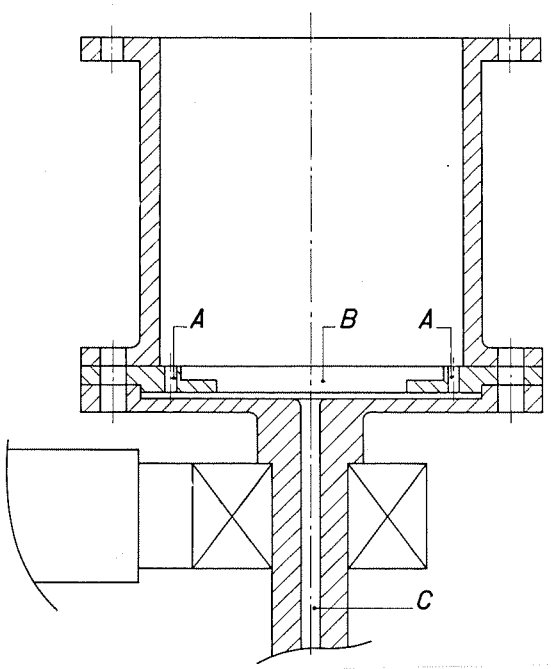


Fig. 8. Cross section through tank of roll-pitch apparatus:

A - 6 outlet orifices ⌀ 4 mm, B - port,
C - drainage well ⌀ 8 mm.

of the tank was given the form of two superimposed plates (Fig. 8). The liquid drained through 6 orifices arranged around the edge of the

upper plate, after which it flowed through a well opening into the middle of the lower plate.

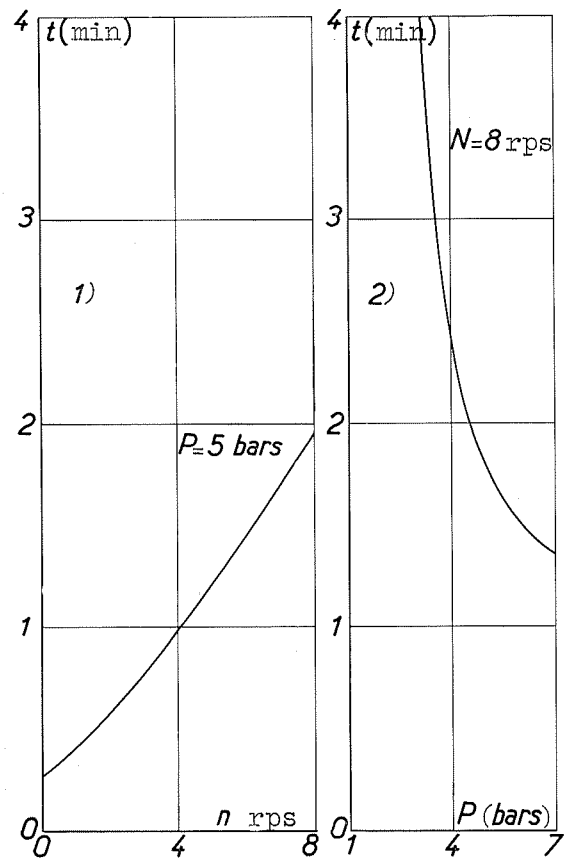


Fig. 9. Emptying time of a column of mercury 200 mm high:

1 - as a function of the rotation frequency; 2 - as a function of the expulsion pressure.

Fig. 9 gives the emptying time for a column of mercury 200 mm high as a function of the rate of rotation of the tank and the expulsion pressure. The high values of t are partly due to the presence

of vortex in the drainage well.

Thus, these few preliminary experiments indicate the need for a more thorough study of the liquid fuel supply systems of missiles subject to axial rotation. /9

VI. CONCLUSIONS

Essentially, this study has brought to light the dangers of axial rotation for a missile containing a liquid mass.

To summarize, in the case of a missile in axial rotation flying in the atmosphere and subject to an aerodynamic restoring couple, there is a risk of resonance not only between the lateral oscillation and the sustained roll of the solid part of the missile but also between this oscillation and the perturbed rotation of the liquid it contains.

There are only three ways of avoiding these effects:

- by giving the missile an axial rotation at takeoff with an angular velocity greater than the maximum critical speed, and maintaining this motion;

- by using missiles in which the geometry of the masses satisfies the sufficient condition of stability and inside which the liquid has no free surface;

- or, finally, by suppressing the rotation of the liquid by means of submerged blades turning in a direction opposite to that of the missile.

The few results obtained on the stabilization of a missile by axial rotation alone suggest that in certain cases this might in fact be possible. On the other hand, however, parasitic effects, such as the vibration of the structure or combustion instabilities, which do not give any particular trouble at the rates of rotation envisaged in the previous case, now threaten to excite motion of the liquid and thus induce resonances equally dangerous for the stability and strength of the missile.

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